

# ANALYSIS AND DESIGN OF FIVE PORT CIRCULAR DISC STRUCTURES FOR SIX-PORT ANALYZERS

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## ABSTRACT

This paper presents a method for designing a five-port disc microstrip circuit suitable for the six-port network analyzers. The method is based on the planar circuit approach in which the two-dimensional Green's function of a circular segment is used. It is found that a bandwidth of about 44% can be obtained (for  $|S_{11}|$  less than 0.2).

## 1. INTRODUCTION

It has been shown [1] that a combination of a symmetrical five port network and a perfect directional coupler possesses the ideal six-port characteristics required for network analyzers. Both circular disc [1,2] and circular ring [1,3,4] configurations having symmetrically located five ports have been proposed. Whereas a five port circular ring structure has been reported in considerable detail, comparatively very little design information is available for a circular disc structure. It may be noted that ease of fabrication makes the circular disc structure more attractive at higher microwave and millimeter wave frequencies.

This paper presents a general method for analyzing five-port circular disc structures. This method uses the planar circuit approach using Green's function for a circular segment with a magnetic wall. Z-matrix characterization of the five-port structure is derived. Numerical results (useful for design purposes) are presented. It is found that a bandwidth of 43.8% (defined for input reflection coefficient less than 0.2) may be obtained without any matching arrangement. Also it is observed that this structure may be used in the higher order (5,1) mode. This leads to five-port operation at higher frequencies for the same disc diameter.

## 2. METHOD OF ANALYSIS

The two-dimensional planar circuit approach is used by replacing the physical disc diameter with an effective diameter surrounded by a perfect magnetic wall.

The impedance matrix of a multiport circular disc is obtained from the available Green's function [5] for a circular segment with a magnetic wall. The elements of the Z-matrix are obtained as follows [5]

$$Z_{ij} = \frac{1}{W_i W_j} \int_{W_i} \int_{W_j} G(s|s_o) ds_o ds \quad (1)$$

where  $W_i$  and  $W_j$  represent the widths of ports  $i$  and  $j$ , respectively, and the Green's function  $G$  for ports on the circumference is given by

$$G(a, \phi | a, \phi_o) = \frac{j\omega\mu d}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma_n \cos\{n(\phi - \phi_o)\}}{(a^2 \frac{n^2}{k_{nm}^2} - k^2) k_{nm}^2} \quad (2)$$

where  $\phi$  and  $\phi_o$  specify the location of the two ports  $i$  and  $j$  as shown in Figure 1a. The substrate parameters are: height  $d$ , permeability  $\mu$  and dielectric constant  $\epsilon_r$ . The parameter  $\sigma_n$  is 1 when  $n=0$  and is equal to 2 otherwise. The effective radius of the disc is 'a' and the wave number  $k$  is  $\omega\sqrt{\mu_o \epsilon_o \epsilon_{rd}}/r$  where  $\epsilon_{rd}$  is the dynamic effective dielectric constant [6] given by

$$\epsilon_{rd} = \frac{C_{dyn}(\epsilon = \epsilon_o \epsilon_r)}{C_{dyn}(\epsilon = \epsilon_o)} \quad (3)$$

where  $C_{dyn}$  represents the dynamic capacitance of the disc.  $Z_{ij}$  is being calculated at frequency  $\omega$ , and  $k_{nm}$  satisfy

$$\frac{\partial}{\partial \rho} J_n(k_{nm} \rho) \Big|_{\rho=a} = 0 \quad (4)$$

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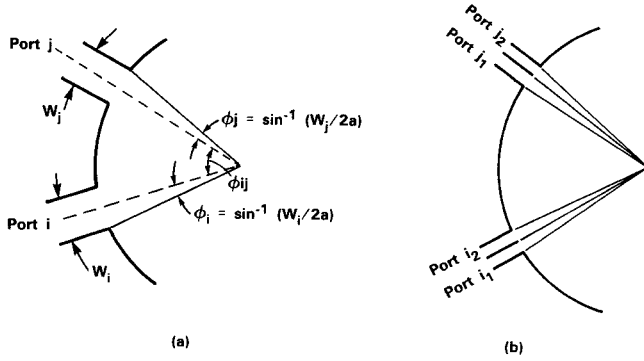


Fig. 1. (a) Parameters for ports at the circumference of a circular disc.  
(b) Division of a wide port into sub-ports.

For any port  $i$ , the  $Z$ -matrix element  $Z_{ii}$  may be written as

$$Z_{ii} = \frac{2j\omega\mu da^2}{\pi W_i^2} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma_n \{1 - \cos[2n \sin^{-1}(W_i/2a)]\}}{n^2 \left(a^2 - \frac{n^2}{k_{nm}^2}\right) (k_{nm}^2 - k^2)} \quad (5)$$

Off-diagonal terms of the impedance matrix are found to be

$$Z_{ij} = \frac{j\omega\mu da^2}{\pi W_i W_j} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \left\{ \frac{\sigma_n}{n^2 \left(a^2 - \frac{n^2}{k_{nm}^2}\right) (k_{nm}^2 - k^2)} \cdot \{ \cos[n(\phi_i - \phi_j)] - \cos[n(\phi_i + \phi_j)] \} \cos(n\phi_{ij}) \right\} \quad (6)$$

Relations (5) and (6) are valid only when the widths  $W_i$  and  $W_j$  are small compared to wavelength ( $\omega\sqrt{\epsilon_r}/c$ ) and therefore any field variation across  $W_i$  and  $W_j$  is negligible. At higher frequencies, or for wide-width ports, more accurate values of  $Z_{ii}$  and  $Z_{ij}$  are obtained by dividing each of the ports  $i$  and  $j$  into two (or more) sub-ports as shown in Figure 1b. The widths of sub-ports  $i_1, i_2, \dots$ , etc., are now small for field variation along the port widths to be negligible. When the sub-ports are being considered, the first step in the  $Z$ -matrix evaluation is to find the  $Z$ -matrix of the network with respect to various sub-ports. This will be a  $10 \times 10$  matrix if there are two sub-ports for each of the five-ports. The  $Z$ -matrix of the five-ports is then derived as

$$[Z_{5p}] = [Y_{5p}]^{-1} \quad (7)$$

Elements of the admittance matrix  $Y_{5p}$  are given by [5]

$$Y_{ij} = \sum_{x=i_1, i_2, \dots} \sum_{y=j_1, j_2, \dots} Y_{xy} \quad (8)$$

where  $Y_{xy}$  are the elements of the admittance matrix of the networks considering each sub-port as an external port.

In the results reported in this paper the frequency variation of  $Z_0$  and the effective width of microstrip lines has been taken into account by using the appropriate dispersion relations [7].

### 3. DESIGN RESULTS

The analytical method discussed above has been used for designing five-port networks in several frequency bands. Some typical results are summarized in Figs. 2 and 3. Figure 2 shows the frequency response of a five-port circuit designed on a 1/16 inch thick substrate ( $\epsilon_r=2.20$ ). In this case the usable frequency range (defined for  $|S_{11}| < 0.2$  or  $VSWR < 1.5$ ) extends from 9.4 GHz to 14.8 GHz which yields a bandwidth of 44.63 percent around 12.1 GHz. By frequency scaling [5] this design can be modified to cover the complete X-band range. It may be noted here, that this frequency range is around the (2,1) mode and the (0,1) mode of the circular disc resonator.

#### Symmetrical 5-port ( $a=8.65$ mm, $t=0.7$ mil, $\epsilon_r=2.2$ )

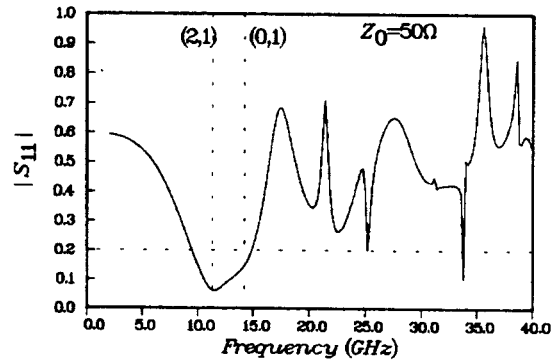


Fig. 2.  $|S_{11}|$  for 5-port circuit on 1/16 inch thick substrate.

Five-port networks may also be designed for operation in a frequency band near the (5,1) mode of the circular disc resonator. Because of the larger physical size and therefore better tolerance characteristics, this design is preferable at millimeter wave frequency. The input reflection coefficient  $|S_{11}|$  for this design (on a 1/32 inch thick substrate) is shown in Fig. 3 for 50  $\Omega$  and 54  $\Omega$  impedances of the outgoing transmission lines. The 50  $\Omega$  circuit has a bandwidth of 3.9 GHz (17.1%) whereas the 54  $\Omega$  circuit has a bandwidth of 4.1 GHz.

The frequency dependence of  $S_{12}$ ,  $S_{13}$  and the differential phase shift between ports 2 and 3 are shown in Fig. 4.

Symmetrical 5-port ( $a=8.65$  mm,  $t=0.7$  mil,  $\epsilon_r=2.2$ )

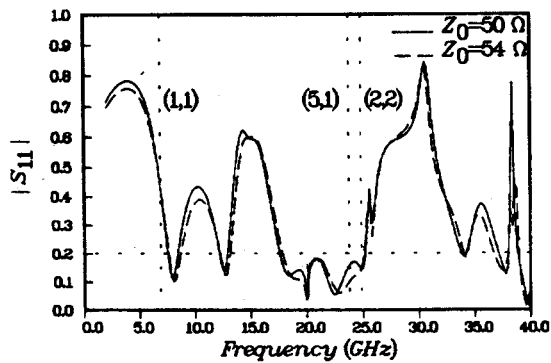


Fig. 3.  $|S_{11}|$  for 5-port circuit on 1/32 inch thick substrate.

Symmetrical 5-port ( $a=8.65$  mm,  $t=1.4$  mil,  $\epsilon_r=2.2$ )

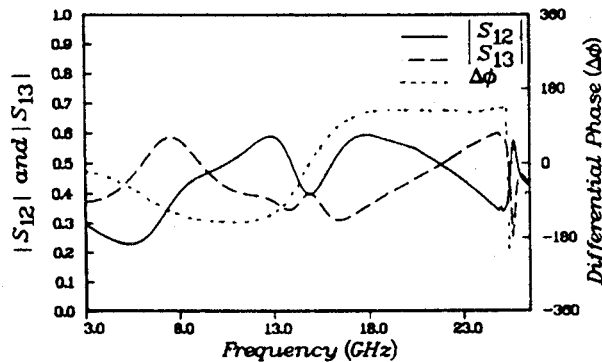


Fig. 4. Transmission coefficients  $|S_{12}|$  and  $|S_{13}|$ , and port-to-port differential phase  $\Delta\phi$  versus the frequency of a symmetrical five-port microstrip circular disc.

#### 4. EXPERIMENTAL RESULTS

Symmetrical five-port circular disc structure designed by using the planar circuit approach discussed above has been fabricated on a 1/32 inch thick dielectric substrate ( $\epsilon_r=2.20$ ). A photograph of this circuit is shown in Fig. 5 and an outline of the circuit is included in Fig. 6.

Extensive measurements have been carried out using an Automatic Network Analyzer and the results are plotted in figures 6 and 7. Good agreement between the theoretical and experimental results confirms the validity of the design approach presented in this paper. The experiments reported above have been carried out with the impedance of the output ports equal to 50  $\Omega$ . Results of the computer-aided analysis has indicated that it is possible to increase the bandwidth of this network by modifying the impedance level of the outgoing ports.

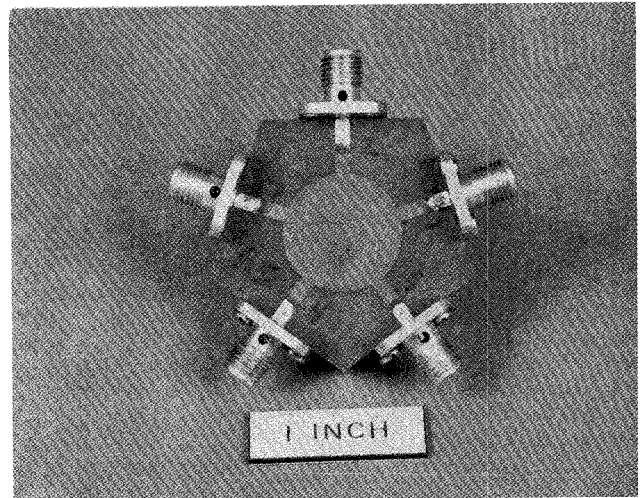


Fig. 5. Photograph of the experimental 5-port microstrip circular disc power divider fabricated on a Duroid substrate.

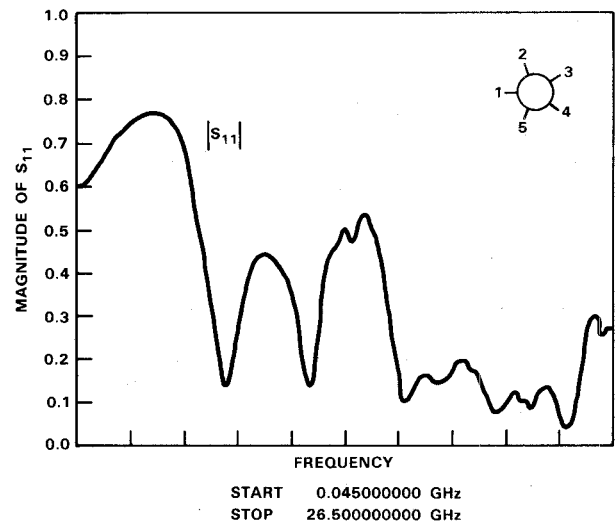


Fig. 6. Experimental behavior of the reflection coefficient  $|S_{11}|$  versus the frequency for a five port circular disc. Substrate material is Duroid.

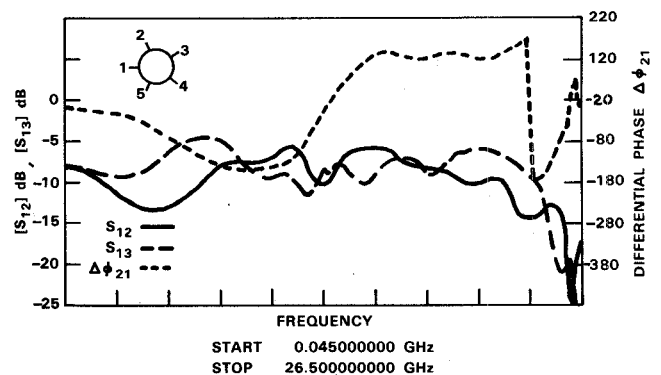


Fig. 7. Measured behavior of the transmission coefficients  $|S_{12}|$  and  $|S_{13}|$ , and the differential phase between port 2 and port 3, versus frequency.

## 5. CONCLUDING REMARKS

A planar circular approach for the analysis and design of a five-port symmetrical circular microstrip disc network has been presented. Experimental results verifying the analytical approach were also presented.

The planar circular approach employed in this paper can be extended to incorporate the effect of the parasitic reactances associated with the junctions of the outgoing microstrip lines and the disc. This may be carried out by considering small lengths of these five outgoing lines as rectangular segments and then use the segmentation method to derive the overall Z-matrix inclusive of the junction reactance effects. This technique can also be extended to incorporate any external impedance transforming or broadbanding circuit elements.

The planar circuit approach described in this paper can also be extended to the analysis and design of a symmetrical five port reduced height waveguide circuit by replacing the Green's function used in this paper by the corresponding Green's function for a circular segment with electric wall around the circumference.

## 6. ACKNOWLEDGMENT

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